Tests for Convergence

- ## Divergence Test:

If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then $\sum_{n=1}^{\infty} a_n$ diverges.

- ## Geometric Series:

A series of the form

$$\sum_{n=0}^{\infty} a(x)^n$$

converges when $|x| < 1$.

- ## Harmonic Series:

The harmonic series

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

diverges.

- ## P-Series:

A series of the form

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

  - converges if $p > 1$
  - diverges if $p \leq 1$.

- ## Integral Test:

Suppose $a_n = f(n)$, where $f(x)$ is decreasing and positive.

  - If $\int_1^{\infty} f(x)\, dx$ converges, then $\sum a_n$ converges.
  - If $\int_1^{\infty} f(x) dx$ diverges, then $\sum a_n$ diverges.

- ## Comparison Test:

Suppose $0 \leq a_n \leq b_n$ for all n above a certain value.

  - If $\sum b_n$ converges, then $\sum a_n$ converges.
  - If $\sum a_n$ diverges, then $\sum b_n$ diverges.

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- ## Limit Comparison Test:

  Suppose $a_n > 0$ and $b_n > 0$ for all n.
  If

  $$\lim_{n\to\infty} \frac{a_n}{b_n} = c, where\ c > 0$$

  Then the two series $\sum a_n$ and $\sum b_n$ either both converge or both diverge.

- ## The Ratio Test:

  Suppose we have a series $\sum a_n$.
  Define

  $$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \mathrm{L}.$$

  - If $\mathrm{L} < 1$, then $\sum a_n$ is convergent.
  - If $\mathrm{L} > 1$, or if L is infinite, then $\sum a_n$ is divergent.
  - If $\mathrm{L} = 1$, then the ratio test is inconclusive.

- ## The Root Test:

  Suppose we have a series $\sum a_n$.
  Define

  $$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \mathrm{L}.$$

  - If $\mathrm{L} < 1$, then $\sum a_n$ is absolutely convergent.
  - If $\mathrm{L} > 1$, then $\sum a_n$ is divergent.
  - If $\mathrm{L} = 1$, then the root test is inconclusive.

- ## Alternating Series Test:

  Suppose we have a series of the form

  $$\sum_{n=1}^{\infty} (-1)^{n-1} * a_n .$$

  This series converges if

  - $0 < a_{n+1} < a_n$ for all n

  and

  - $\lim_{n\to\infty} a_n = 0$