Properties of Logarithms

$log_b(x) = y$ is equivalent to $x = b^y$

Common logarithm: $\log x = log_{10}x$

Natural logarithm: $ln x = log_e x$

Basic Properties of Logarithms

Let $b > 0$ with $b \neq 1$.

1. $log_b(b) = 1$
2. $ln(e) = 1$
3. $log_b(1) = 0$
4. $log_b(b^x) = x$
5. $b^{log_b(x)} = x$

Properties of Logarithms

Let M, N be positive real numbers.

1. If $M = N$, then $log_b(M) = log_b(N)$
2. $log_b(M \cdot N) = log_b(M) + log_b(N)$
3. $log_b\left(\frac{M}{N}\right) = log_b(M) - log_b(N)$
4. $log_b(M^k) = k \cdot log_b(M)$

Properties of Exponents

Let M, N be real numbers.

1. If $M = N$, then $b^M = b^N$
2. $b^{M+N} = (b^M) \cdot (b^N)$
3. $b^{M-N} = \frac{b^M}{b^N}$
4. $(b^M)^N = b^{M \cdot N} = (b^N)^M$

Change of Base Formula

Let $a, b > 0$ with $a, b \neq 1$.

$$log_b(x) = \frac{log_a(x)}{log_a(b)}$$

OCCC Math Lab

Examples of Logarithms

1) **Write $2^y = x$ in logarithmic notation.**

Note: $b^y = x$ is equivalent to $log_b(x) = y$.
$2^y = x$ in logarithmic notation is
$log_2(x) = y$.

2) **Evaluate $log_2(8)$.**

Note: $8 = 2^3$.

$$log_2(8) = log_2(2^3) = 3 \cdot log_2(2) = 3 \cdot 1 = 3$$

Thus,

$$log_2(8) = 3.$$

3) **Evaluate $ln(e^5)$.**

Note: $ln(x) = log_e(x)$.

$$\ln(e^5) = log_e(e^5) = 5 \cdot log_e(e) = 5 \cdot 1 = 5$$

Thus,

$$ln(e^5) = 5.$$

4) **Rewrite $log_3(x^6 \cdot z^2)$ as a sum, difference, or product of logarithms, and simplify if possible.**

$$log_3(x^6 \cdot z^2) = log_3(x^6) + log_3(z^2)$$
$$= 6 \cdot log_3(x) + 2 \cdot log_3(z).$$

Thus,

$$log_3(x^6 \cdot z^2) = 6 \cdot log_3(x) + 2 \cdot log_3(z).$$

5) **Solve $6+3 \cdot log_2(x)=12$ by writing the equation in exponential form.**

$$6 + 3 \cdot log_4(x) = 12$$
$\Rightarrow$ $$3 \cdot log_4(x) = 6$$
$\Rightarrow$ $$log_4(x) = 2$$
$\Rightarrow$ $$x = 4^2$$
$\Rightarrow$ $$x = 16$$