

# Laplace Transforms

$y(t) = \mathcal{L}^{-1}[Y(s)]$	$Y(s) = \mathcal{L}[y(t)]$
$y(t)$	$Y(s) = \int_0^{\infty} y(t)e^{-st} dt$
$y(t) = e^{at}$	$Y(s) = \frac{1}{s-a} \quad (s > a)$
$y(t) = t^n$	$Y(s) = \frac{n!}{s^{n+1}} \quad (s > 0)$
$y(t) = \sin(\omega t)$	$Y(s) = \frac{\omega}{s^2 + \omega^2}$
$y(t) = \cos(\omega t)$	$Y(s) = \frac{s}{s^2 + \omega^2}$
$y(t) = e^{at} \cdot \sin(\omega t)$	$Y(s) = \frac{\omega}{(s-a)^2 + \omega^2}$
$y(t) = e^{at} \cdot \cos(\omega t)$	$Y(s) = \frac{(s-a)}{(s-a)^2 + \omega^2}$
$y(t) = t \cdot \sin(\omega t)$	$Y(s) = \frac{2\omega s}{(s^2 + \omega^2)^2}$
$y(t) = t \cdot \cos(\omega t)$	$Y(s) = \frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
$y(t) = u_a(t)$	$Y(s) = \frac{e^{-as}}{s} \quad (s > 0)$
$y(t) = \delta_a(t)$	$Y(s) = e^{-as}$

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Rules for Laplace	Rules for Inverse Laplace
$\mathcal{L}[y(t)] = Y(s)$	$\mathcal{L}^{-1}[Y(s)] = y(t)$
$\mathcal{L}[w(t)] = W(s)$	$\mathcal{L}^{-1}[W(s)] = w(t)$
$\mathcal{L}\left[\frac{dy}{dt}\right] = s\mathcal{L}[y] - y(0)$	
$\mathcal{L}[y + w] = \mathcal{L}[y] + \mathcal{L}[w]$	$\mathcal{L}^{-1}[Y + W] = \mathcal{L}^{-1}[Y] + \mathcal{L}^{-1}[W]$
$\mathcal{L}[\alpha \cdot y] = \alpha \cdot \mathcal{L}[y] = \alpha \cdot Y(s)$	$\mathcal{L}^{-1}[\alpha \cdot Y] = \alpha \cdot \mathcal{L}^{-1}[Y] = \alpha \cdot y(t)$
$\mathcal{L}[u_a(t) \cdot y(t - a)] = e^{-as}\mathcal{L}[y]$	$\mathcal{L}^{-1}[e^{-as}Y(s)] = u_a(t) \cdot y(t - a)$
$\mathcal{L}[e^{at}y(t)] = Y(s - a)$	$\mathcal{L}^{-1}[Y(s - a)] = e^{at}y(t)$
<b><u>Convolutions</u></b>	
$(f * g)(t) = \int_0^t f(t - u) \cdot g(u) du$	
$\mathcal{L}[(f * g)] = \mathcal{L}[f] \cdot \mathcal{L}[g] = F(s) \cdot G(s)$	$\mathcal{L}^{-1}[F(s) \cdot G(s)] = \mathcal{L}^{-1}[F] \cdot \mathcal{L}^{-1}[G]$