Elasticity

We have looked at how market agents interact with demand and supply, and we have shown how to find an equilibrium price and quantity given functional forms for supply and demand. We further showed that at equilibrium, a decrease in supply, ceteris paribus, led to a decrease in Q and an increase in P. An important point is that firm agents in the economy want to know how a price change will effect quantity demanded or supplied. The effect can be show as follows:

Suppose there is a shift in supply that increases price and decrease quantity. How does the change affect Total Revenue \( (P \times Q) \)? This depends upon how much the quantity demanded changes with the change in price.

Suppose

\[
\begin{align*}
Q_0 &= 10 & P_0 &= 3 & TR &= 30 \\
Q_1 &= 5 & P_1 &= 8 & TR &= 40 \\
\Delta TR &= +10
\end{align*}
\]

Instead, suppose

\[
\begin{align*}
Q_0 &= 10 & P_0 &= 3 & TR &= 30 \\
Q_1 &= 5 & P_1 &= 6 & TR &= 30 \\
\Delta TR &= 0
\end{align*}
\]

Notice how different changes in quantity demanded in response to a price change can alter whether revenues increase, decrease, or stay the same. The absolute change in TR can be measured, but the slope of demand depends upon the units in which the goods are measured, and thus it is difficult to compare demand across different goods. To avoid this
problem, we look for a measure of how price affects demand and TR which is independent of specific good units.

## Price Elasticity of Demand

Price Elasticity of Demand: Measures the response of quantity demanded of a good to a change in its price.

We measure elasticity of demand by the following formula:

$$\varepsilon_p^d = -\frac{\%\Delta Q^d}{\%\Delta P^d}$$

Note: that $\varepsilon_p^d$ is independent of units, but how is that?
$\varepsilon_p^d$ says that quantity demanded will respond to a price change as follows:

$$\%\Delta Q^d = \frac{Q_1 - Q_0}{Q} = \frac{\Delta Q}{Q} \quad (1)$$

$$\%\Delta P^d = \frac{P_1 - P_0}{P} = \frac{\Delta P}{P} \quad (2)$$

So, we see that:

$$\varepsilon_p^d = -\frac{\%\Delta Q^d}{\%\Delta P^d} = -\frac{\Delta Q^d}{\Delta P^d}(\frac{P}{Q})$$

Thus, we see that $\varepsilon_p^d$ is an unitless measurement, because the goods units and price units cancel each other out.

So, if $Q_0 = 10$ $P_0 = 3$ and $Q_1 = 6$ and $P_1 = 5$ then

$$\varepsilon_p^d = -\left(-\frac{4}{2}\right)\left(\frac{4}{8}\right) = 1$$

So, if $Q_0 = 10$ $P_0 = 2$ and $Q_1 = 6$ and $P_1 = 5$ then

$$\varepsilon_p^d = -\left(-\frac{4}{3}\right)\left(\frac{7}{16}\right) = \frac{7}{12}$$

So, if $Q_0 = 10$ $P_0 = 4$ and $Q_1 = 6$ and $P_1 = 5$ then

$$\varepsilon_p^d = -\left(-\frac{4}{1}\right)\left(\frac{9}{16}\right) = \frac{9}{4}$$

We use average price and quantity because there would otherwise be two different elasticities, depending on how price changes, i.e. does it rise or does it drop.

We note that the price level rising led to a decrease in quantity demanded, due to the negative relationship between price and quantity demanded, so we include the negative sign to make $\varepsilon_p^d$ an absolute value for elasticity.

What if price drops?
So, if \( Q_0 = 6 \ P_0 = 5 \) and \( Q_1 = 10 \ P_1 = 4 \) then 
\[
\varepsilon_d^p = -\left(-\frac{4}{1}\right)(\frac{9}{16}) = \frac{9}{4},
\]
and we get the same result no matter how price changes.

When price rises, ceteris paribus, quantity demanded will drop and if price drops then quantity demanded rises. Therefore the negative relationship between price and quantity demanded is what causes \( \varepsilon_d^p \) to always be negative. Therefore, we can use the absolute value of elasticity as a means of comparison.

With a measure of elasticity, we can compare how demand reacts to price, and we can compare elasticities across goods. But how do we know how “large” an elasticity actually is.

We define the following conventions for comparison

\[
\begin{align*}
\varepsilon_d^p = 0 & \Rightarrow \text{Perfect Elasticity} \\
0 \leq \varepsilon_d^p \leq 1 & \Rightarrow \text{Relative Inelasticity} \\
\varepsilon_d^p = 1 & \Rightarrow \text{Unitary Elasticity} \\
\varepsilon_d^p > 1 & \Rightarrow \text{Relative Elasticity} \\
\varepsilon_d^p = \infty & \Rightarrow \text{Perfect Inelasticity}
\end{align*}
\]

How does \( \varepsilon_d^p = 0 \) or \( \varepsilon_d^p = \infty \)?

\[
\varepsilon_d^p = \left(\frac{\Delta Q}{\Delta P}\right)\left(\frac{P}{Q}\right) = \infty
\]

It does not make much sense that \( Q = 0 \), so \( \Delta P \) must equal zero. How does that look on a graph (see \( D_1 \) below).

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\[
\varepsilon_d^p = \left(\frac{\Delta Q}{\Delta P}\right)\left(\frac{P}{Q}\right) = 0
\]

It does not make much sense that \( P = 0 \), so \( \Delta Q \) must equal zero (see \( D_2 \) above).

Three things can affect the elasticity of demand:

1. Number and ease of substitutes for the good.

   - Ex: Doctors don’t have a very good and easy to find substitute (that is healthy)
   - Ex: Apples have easy to find substitutes, e.g. oranges, grapefruit, grapes, etc.
   - Note: Substitutability depends upon the level that you inspect the good. Health care may have a relatively more elastic demand curve than the demand for neurosurgeons themselves. The demand for apples may be more elastic than the demand for snack food in general.
2. Amount of Income spent on the good

The more relative income spent on the good, the higher the elasticity. This is because a small rise in price can take a big bite out of the household’s budget and cause individuals to rethink how they are going to spend their money, i.e. look for substitutes.

Again, we can use our doctor example. if doctors raise their price by ten dollars, you hardly change the amount of doctors services used, whereas if apples double in price, you will definitely buy less apples.

3. The time frame of the price change. Generally given more time, the more elastic demand is. This is because it is more likely that a substitute can be found for the good as time passes. Thus, we ask ourselves how time may be specified within demand:

- Short-Run Demand: The demand curve that describes the initial response to a price change, ceteris paribus. This response is a measure of how people feel about the price change, among other things. If they view a price rise as temporary, then demand may drop or if a price drop is viewed as temporary then demand may rise. The SR demand holds everything else constant, i.e. technology, supply, money, interest rates, prices of other goods, etc.

- Long-Run Demand: The LR demand measures a response to price after all other adjustments have taken place, i.e. prices of other goods, etc. LR demand tends to be more relatively elastic than SR demand.

Income Elasticity of Demand  \( \epsilon^m_d \)

\( \epsilon^m_d \) measures how demand can change as income is changed. This depends upon the

\[
\epsilon^m_d = \frac{\%\Delta Q^d}{\%\Delta M}
\]

\( \epsilon^m_d \) is unlike \( \epsilon^p_d \) in that it can be a negative value.

\[
\begin{align*}
\epsilon^m_d &> 1 \quad \Rightarrow \quad \text{Elastic} \\
0 < \epsilon^m_d < 1 &\quad \Rightarrow \quad \text{Inelastic} \\
\epsilon^m_d &< 0 \quad \Rightarrow \quad \text{Negative Income Elasticity}
\end{align*}
\]

A normal good as a good which is demanded in greater quantities as income increases. Thus, (1) and (2) represent normal goods, i.e. \( \epsilon^m_d > 0 \). If \( \epsilon^m_d < 0 \), then demand is decreasing as income is increasing, and the good is an inferior good. Normal goods are goods such as automobiles, jewelry, steak, imported beer, and inferior goods are goods such as hamburger, bus rides, used cars, etc.

Cross Elasticity of Demand

Recall that we defined goods in relation to each other as complements or substitutes, where complements are goods consumed together and substitutes are goods consumed in place of
each other. We can look at how demand changes when the price of related goods increases. The Cross Elasticity of Demand

$$
\varepsilon^{x_d} = \frac{\% \Delta Q^d}{\% \Delta P^x}
$$

Note: $\varepsilon^{x_d} > 0$ implies that the two goods are substitutes, $< 0$ implies the two goods are complements.

The more easily goods are complements or substitutes, the higher their relative cross-elasticities are.

**Elasticity of Supply $\varepsilon^p_s$**

Before, we looked at how a shift in the supply curve which would cause a movement along the demand curve. Now we want to look at how a change in demand can cause a movement along the supply curve. To know how price and quantity are affected by a change in demand, we need to know the elasticity of supply:

$$
\varepsilon^p_s = \frac{\% \Delta Q^s}{\% \Delta P^s} = \frac{\Delta Q^s}{\Delta P^s} \frac{P}{Q}
$$

Two things can effect $\varepsilon^p_s$: Technology at time of price change and the time frame of the price change.

- **Technology**: Some goods have impossible technology, e.g. The Chrysler Building. It is impossible to replicate it exactly, so $\varepsilon^p_s = 0$. Some have easy technology such as gravel production, which has a constant cost of production and thus has elastic supply. Most goods have $0 < \varepsilon^p_s < 1$.

- **Time**: We discussed two time periods for demand, we can also specify three time periods for supply.

  1. **Momentary Supply**: The response immediately after a price change. Some foods have perfectly inelastic supply, as well as some durable goods that have a set production process like prefab cement, etc. This is because immediate supply cannot adjust due to production processes. Other goods, such as electricity, water and such have elastic momentary supply. During peak hours, demand is high but the price does not change with the quantity demanded.

  2. **Short Run Supply**: SR supply represents a time frame when part of the technological process can change. An easy adjustment is the amount of labor and short-term capital such as rental equipment. Relatively inelastic compared to LR supply, but more elastic than MS.

  3. **Long Run Supply**: LR supply is the supply after all possible aspects of technology have been allowed to adjust. This would include new factories and assembly plants. Development of R-n-D processes. Typically this process can take years to develop, as in current computer technology. The first computers were available
in the 1960’s, but it has taken over 25 years for the technological process to get us where we are today with personal computers.

LR supply is relatively more elastic than SR supply and MS.

So, we can draw a typical series of supply curves. Each SR supply curve is relatively more elastic.