An arc of a circle is a "portion" of the circumference of the circle. The length of an arc is simply the length of its "portion" of the circumference. Actually, the circumference itself can be considered an arc length. The length of an arc (or arc length) is traditionally symbolized by $s$. The radian measure of a central angle of a circle is defined as the ratio of the length of the arc the angle subtends, $s$, divided by the radius of the circle, $r$. 

$\theta = \frac{s}{r}$

Angular Measure

The position of an object can be described using polar coordinates—$r$ and $\theta$—rather than $x$ and $y$. The figure at left gives the conversion between the two descriptions.
It is most convenient to measure the angle $\theta$ in radians:

\[ s = r\theta \]  

Angular Measure

The small-angle approximation is very useful, as it allows the substitution of $\theta$ for $\sin \theta$ when the angle is sufficiently small.

Angular Measure

Quick Question 1

- When you are watching the NASCAR Daytona 500, the 5.5m long race car subtends an angle of 0.31 degrees. What is the distance from the race car to you?

Quick Question 1 Solution

In analogy to the linear case, we define the average and instantaneous angular speed:

\[ \omega = \frac{\Delta \theta}{\Delta t} = \frac{\theta - \theta_0}{t - t_0} \]

\[ \omega = \frac{\theta}{t} \quad \text{or} \quad \theta = \omega t \]

Angular Speed & Velocity

The direction of the angular velocity is along the axis of rotation, and is given by a right-hand rule.

$\omega$ is positive for a counter-clockwise rotation. $\omega$ is negative for a clockwise rotation.
Relationship between tangential and angular speeds:

\[ \vec{v} = r\omega \]

This means that parts of a rotating object farther from the axis of rotation move faster.

The period is the time it takes for one rotation; the frequency is the number of rotations per second.

\[ f = \frac{1}{T} \]

The relation of the frequency to the angular speed:

\[ \omega = \frac{2\pi}{T} = 2\pi f \]

Quick Question 2

- A bicycle wheel rotates uniformly through 2.0 revolutions in 4.0 s.
  
  (A) What is the average angular speed of the wheel?
  
  (B) What is the tangential speed of a point 0.10 m from the center of the wheel?
  
  (C) What is the period?
  
  (D) What is the frequency?

Quick Question 2.b

- A “weed eater” trimmer is a tool for cutting grass and weeds. It utilizes a nylon or plastic “string” that rotates about an axis perpendicular to one end of the string. The string rotates at 47 rev/s, and its tip has a tangential speed of 54 m/s. What is the length of the rotating string?

Centripetal Acceleration

A careful look at the change in the velocity vector of an object moving in a circle at constant speed shows that the acceleration is toward the center of the circle.

The same analysis shows that the centripetal acceleration is given by:

\[ a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2 \]
Quick Question 3

The aorta is a major artery, rising upward from the left ventricle of the heart and curving down to carry blood to the abdomen and lower half of the body. The curved artery can be approximated as a semicircular arch whose diameter is 5.0 cm. If blood flows through the aortic arch at a speed of 0.32 m/s, what is the blood’s centripetal acceleration?

\[
a_c = \frac{v^2}{r} = \frac{(0.32 \text{ m/s})^2}{0.025 \text{ m}} = 4.1 \text{ m/s}^2
\]

Quick Question 4:

At NASA, astronauts are placed in large centrifuge that moves in a circular path with a radius of 15 m. If the astronauts in this centrifuge experience a centripetal acceleration 9.0 times that of gravity, what is the linear speed of the capsule?

Quick Question 5

A steel ball is tied to the end of a string and swung in a vertical circle at constant speed. Complete the following statement: The direction of the acceleration of the ball is always...

a) perpendicular to the circle.

b) toward the center of the circle.

c) tangent to the circle.

d) radially outward from the circle.

e) vertically downward.

Quick Question 6

A bicycle racer is traveling at constant speed \( v \) around a circular track. The centripetal acceleration of the bicycle is \( a_c \). What happens to the centripetal acceleration of the bicycle if the speed is doubled to \( 2v \)?

a) The centripetal acceleration increases to \( 4a_c \).

b) The centripetal acceleration decreases to \( 0.25a_c \).

c) The centripetal acceleration increases to \( 2a_c \).

d) The centripetal acceleration decreases to \( 0.5a_c \).

e) The centripetal acceleration does not change.
Centripetal Acceleration

- “Center seeking” always points toward center of circle
- Depends on velocity and radius of circle
- Never affects magnitude (speed) of velocity, only affects direction

Another acceleration causes the change in direction & keeps object moving in a circle.

\[ a_c = \frac{v^2}{r} \]

Quick Question 7:
A satellite travels at a constant speed in a circular orbit around a planet. Which of the following arrows correctly indicates the direction of the net force on the satellite at the instant shown in the drawing?

Quick Question 8
A satellite travels at a constant speed in a circular orbit around a planet. Which of the following arrows correctly indicates the direction of the net force on the satellite at the instant shown in the drawing?

Quick Question 9
A 0.015-kg ball is shot from the plunger of a pinball machine. Because of a centripetal force of 0.028 N, the ball follows a circular arc whose radius is 0.25 m. What is the speed of the ball?

\[ v = \sqrt{\frac{F_c r}{m}} = \sqrt{\frac{(0.028 \text{ N})(0.25 \text{ m})}{0.015 \text{ kg}}} = 0.68 \text{ m/s} \]

Quick Question 10
A block is hung by a string from the inside roof of a van. When the van goes straight ahead at a speed of 28 m/s, the block hangs vertically down. But when the van maintains this same speed around an unbanked curve (radius 150 m), the block swings toward the outside of the curve. Then the string makes an angle \( u \) with the vertical. Find \( u \).
Quick Question 10 ...

The centripetal acceleration of the block is:

\[ a_c = \frac{v^2}{r} = \frac{(28 \text{ m/s})^2}{150 \text{ m}} = 6.2 \text{ m/s}^2 \]

The angle can be obtained from:

\[ \tan \theta = \frac{a_c}{g} \]

Quick Question 11 ...

The blade of a windshield wiper moves through an angle of 90.08° in 0.40 s. The tip of the blade moves on the arc of a circle that has a radius of 0.45 m. What is the magnitude of the centripetal acceleration of the tip of the blade?

\[ v = \frac{1}{4}(2\pi r)/t = \pi r / (2t) \]

\[ a_c = \frac{v^2}{r} = \frac{(\pi r / 2t)^2}{r} = \frac{\pi^2 r}{4t^2} \]

Angular Acceleration

The average angular acceleration is the rate at which the angular speed changes:

\[ \bar{\alpha} = \frac{\Delta \omega}{\Delta t} \]

In analogy to constant linear acceleration:

\[ \alpha = \frac{\omega - \omega_0}{t} \]

\[ \omega = \omega_0 + \alpha t \]

Angular Acceleration

Don’t confuse the angular acceleration with the centripetal acceleration......

....angular acceleration indicates how rapidly the angular velocity is changing.

...centripetal acceleration is a vector quantity that points toward the center of a particle’s circular path.

If the angular speed is changing, the linear speed must be changing as well. The tangential acceleration is related to the angular acceleration:

\[ a_t = \frac{\Delta v}{\Delta t} = \frac{\Delta (r \omega)}{\Delta t} = \frac{r \Delta \omega}{\Delta t} = r \alpha \]
Angular Acceleration

**Quick Question 12**

The disk in a computer hard-drive spins up to 5400 rpm in 2.00 s. What is the angular acceleration of the disk? At the end of 2.00 s, how many revolutions has the disk made?

The initial angular velocity is \( \omega_i = 0 \) rad/s. The final angular velocity is \( \omega_f = 5400 \) rpm. This value needs to be converted to the correct units before using the equations for angular acceleration and angular displacement during this acceleration. Recall that each revolution corresponds to an angular displacement of \( \pi \).

**Quick Question 13**

- A wheel is rotating with a constant angular acceleration of 3.5 rad/s\(^2\). If the initial angular velocity is 2.0 rad/s and is speeding up, find:
  
  (A) the angle the wheel rotates through in 2.0s.
  
  (B) the angular speed at t=2.0s.