

Using ZERO to Approximate Solutions of an Equation

Instructions for the TI-84 Plus

Example: Find the solutions of the equation $x^3 - x^2 = 2x - 1$ by using ZERO. Round the solutions to the nearest hundredth.

The instructions in this help sheet assume that you are familiar with graphing. If you need to review, please work through the help sheets *Preparing the Calculator for Graphing* and *Graphing an Equation*.

The first step is to set the equation equal to zero, so transform $x^3 - x^2 = 2x - 1$ into the equivalent equation $x^3 - x^2 - 2x + 1 = 0$. Enter the function editor by pressing $\boxed{Y=}$. Make sure that all functions are cleared out. Position your cursor to the right of Y_1 . Enter the expression $x^3 - x^2 - 2x + 1$ by pressing $\boxed{X,T,\theta,n}$ $\boxed{\wedge}$ $\boxed{3}$ $\boxed{-}$ $\boxed{X,T,\theta,n}$ $\boxed{x^2}$ $\boxed{-}$ $\boxed{2}$ $\boxed{X,T,\theta,n}$ $\boxed{+}$ $\boxed{1}$ \boxed{ENTER} . The function editor should look like the one shown in figure 1. To adjust your graphing window to the standard graphing screen and to graph Y_1 , press \boxed{ZOOM} $\boxed{6}$. This will result in the graph shown in figure 2.

Fig. 1

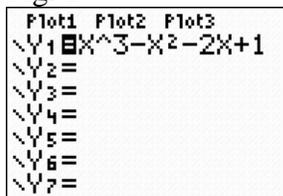
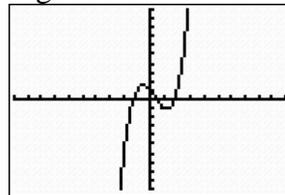


Fig. 2



A zero of $f(x)$ is an x -value that will make the function evaluate to zero when you substitute that value in place of x . Graphically, a zero of $f(x)$ occurs at the x -value where the function intersects the x -axis. There are three solutions to the equation

$x^3 - x^2 - 2x + 1 = 0$ as shown in figure 2. To find the first solution, press $\boxed{2nd}$ \boxed{TRACE} .

This will take you to the calculate menu as shown in figure 3. Press $\boxed{2}$ to select **ZERO**. The result is shown in figure 4.

Fig. 3

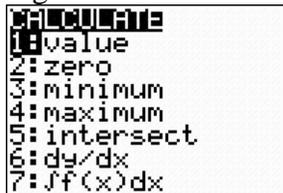
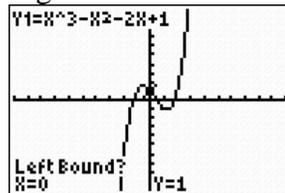


Fig. 4



The calculator needs some boundaries to know which zero you want. It will ask you for a left bound (a point with an x -value less than the zero), a right bound (a point with an x -value greater than the zero), and a guess (a point near the zero). It is important that there is only one zero contained between the left bound and right bound that you pick.

Let's find the leftmost zero first. To do this press  until the cursor is to the left of the zero. In this example, the cursor will be below the x -axis. Press  to set the left bound. Now press  until the cursor is to the right of the zero but is still before the next zero. In this example, the cursor will be above the x -axis. Press  to set the right bound. The calculator now prompts you to Guess. Place your cursor near the zero and press . This process is illustrated in figures 4 through 7. Figure 7 shows that the zero is -1.25 , so $x = -1.25$ is a solution to $x^3 - x^2 = 2x - 1$.

Fig. 5

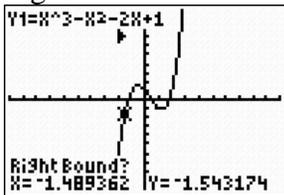


Fig. 6

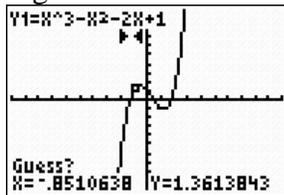
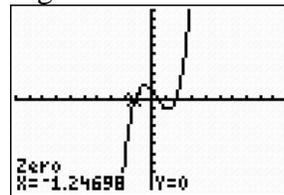


Fig. 7



Next, let's find the centermost zero. You will repeat the same process. Press   to enter the CALCULATE menu (as shown in figure 8). Press  to select ZERO. The result is shown in figure 9.

Fig. 8

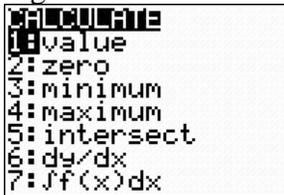


Fig. 9

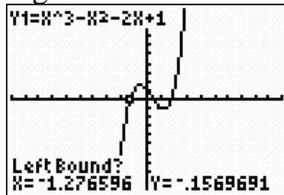
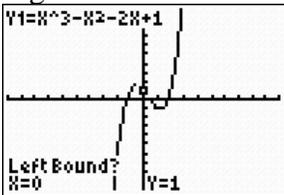
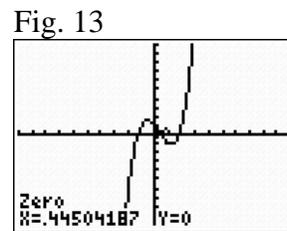
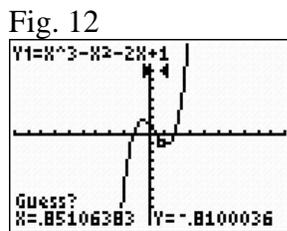
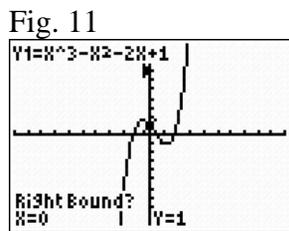


Fig. 10

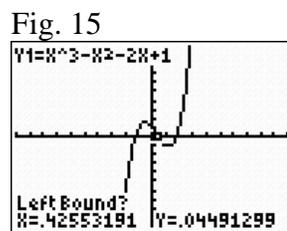
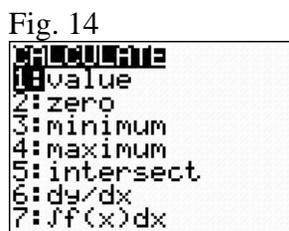


To find the centermost zero, move the cursor to the right by pressing  until the cursor is past the first zero and to the left of the center zero. In this example, the cursor will be above the x -axis as shown in figure 10. Press  to set the left bound. The calculator will request the right bound. The result is shown in figure 11. Move the cursor to the right by pressing  until the cursor is to the right of the center zero but is still before the last zero. In this example, the cursor will be below the x -axis. Press  to set the right bound. The calculator will request your guess. The result is shown in

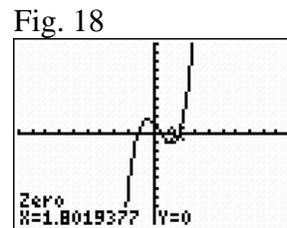
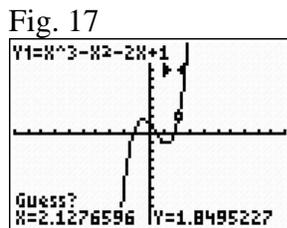
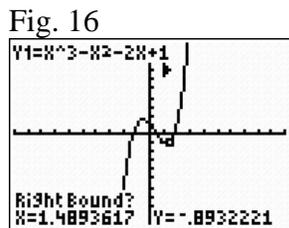
figure 12. Press **ENTER** again to tell the calculator that you are close to the zero. Figure 13 shows that the zero is 0.45, so $x = 0.45$ is a solution to $x^3 - x^2 = 2x - 1$.



Finally, let's find the third solution to the equation $x^3 - x^2 = 2x - 1$. You will repeat the same process as you did to find the first and second solutions. Press **2nd** **TRACE**. This will take you to the calculate menu as shown in figure 14. Press **2** to select **ZERO**. The result is shown in figure 15.



Move the cursor toward the right most solution by pressing **→**. Press **ENTER** while your cursor is still to the left of the zero. This will mark the left bound. The calculator will request the right bound. The result is shown in figure 16. Next, move the cursor to the right of the zero by pressing **→**. In this example, your cursor will be above the x -axis, press **ENTER** to set the right bound. The calculator will request your guess as shown in figure 17. Press **ENTER** again to tell the calculator that you are close to the zero. Figure 18 shows that the zero is 1.80, so $x = 1.80$ is a solution to $x^3 - x^2 = 2x - 1$.



The three solutions, rounded to the nearest hundredth, to the equation $x^3 - x^2 = 2x - 1$ are $x = -1.25$, 0.45 , and 1.80 .