# Rational Zeros of a Polynomial

Instructions for the TI-84 Plus

**Example:** Find all the rational zeros for $f(x) = 10x^3 + 11x^2 - 20x - 21$ and factor $f(x)$ into linear factors.

The first step in any factoring problem is to factor out any common factors. There are no factors that are common to all four terms (other than 1 or $-1$), so we move on to the next step.

The function $f(x)$ is a polynomial with integer coefficients, so the Rational Zeros Theorem tells us that all of the rational zeros of $f(x)$ can be written with a factor of 21 (the absolute value of the constant term) as the numerator and a factor of 10 (the absolute value of the leading coefficient) in the denominator.
The positive factors of 21 are 1, 3, 7, and 21.
The positive factors of 10 are 1, 2, 5, and 10.
Therefore, the possible rational zeros for $f(x)$ are:
$\frac{1}{1}, \frac{3}{1}, \frac{7}{1}, \frac{21}{1}, \frac{1}{2}, \frac{3}{2}, \frac{7}{2}, \frac{21}{2}, \frac{1}{5}, \frac{3}{5}, \frac{7}{5}, \frac{21}{5}, \frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{21}{10}$ and the opposites of these numbers.

The easiest way to find out which of these **possible** rational zeros are **actual** zeros of $f(x)$ is to use the Remainder Theorem. The Remainder Theorem tells us that if we substitute a number, call it *n*, into $f(x)$, the result will be the same as the remainder when we divide $f(x)$ by $x - n$. If *n* is a factor of $f(x)$, then the remainder will be zero. All this means that if we substitute all these **possible** rational zeros into $f(x)$, those that result in an answer of zero are **actual** zeros of $f(x)$.

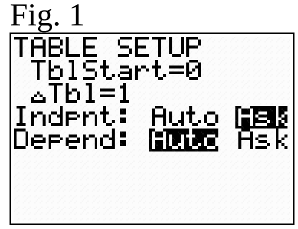
The calculator can make the work of substituting the possible rational zeros into $f(x)$ much easier. For more detailed instructions on the use of a table, look at the help sheet *Creating a Table.*

## Setting the Table to ASK mode

We want to tell the calculator to ASK us which values we want for the independent variable (*x*) then automatically calculate the dependent variable ($f(x)$ or $y$) that corresponds to the *x*-value.

To access the Table Setup menu, press [2nd] then [WINDOW]. Use the arrow keys to position your cursor on the word ASK beside Independent (Indpnt) then press [ENTER]. Use the arrow keys to position your cursor on the word AUTO beside Dependent (Depend) then press [ENTER]. The values for Table Start (TblStart) and Table Increment ($\Delta$Tbl) are not important in ASK mode, so it does not matter which numbers are there. The rest of your screen should look like figure 1.

Fig. 1



## Testing Possible Rational Zeros and Writing in Linear Factors

To enter $f(x)$ into the function editor, press Y=. Use the CLEAR key to erase any equations that are already in the calculator. Use the arrow keys to position your cursor to the right of the equal sign beside $Y_1$ and enter the equation. Your window should look similar to figure 2. (Entering an equation is covered more thoroughly in the help sheet *Graphing an Equation*.)

Fig. 2

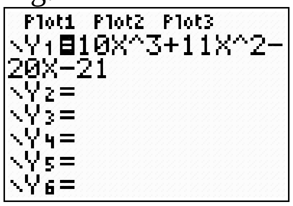



Fig. 3

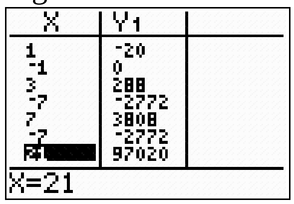



Fig. 4

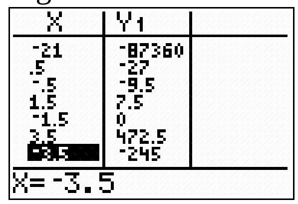



Fig. 5

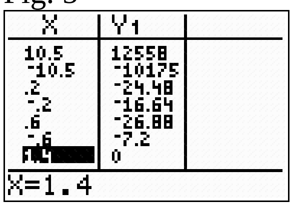



Fig. 6

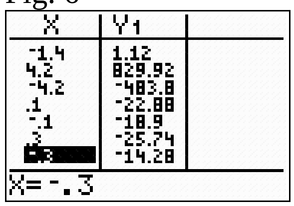
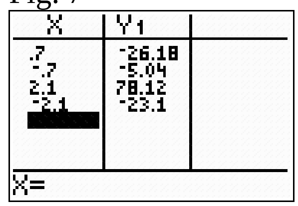



Fig. 7




To access the Table menu, press 2nd then GRAPH. If your screen is not blank, use the DEL key to erase the existing numbers or just type over them. Start entering the numbers from your list of **possible** rational zeros. Don't forget to enter the negative numbers as well as the positive numbers. When you enter fractions, just use the division sign. For example, to enter $-\frac{21}{5}$, press (−) 2 1 ÷ 5. The results for all of these values are listed in figures 3 through 7. By looking at these screens, we see that the possible rational zeros $-1$, $-\frac{3}{2}$, and $\frac{7}{5}$ (represented by the decimals $-1$, $-1.5$, and $1.4$) resulted in a value of 0 when they were substituted into $f(x)$. This means that these three values are zeros of $f(x)$. We know that we have at most three real zeros because $f(x)$ is a polynomial of degree three. We found all three zeros, so it was not even necessary to compute the values for these last two screens. The Factor Theorem tells us that if a number, *n*, is a zero of a polynomial, then $x-n$ is a factor of that polynomial. There were no common factors (other than 1 or $-1$); therefore, $f(x)$ can be written as $(x-{}^{-}1)\left(x-{}^{-}\frac{3}{2}\right)\left(x-\frac{7}{5}\right)$. One simplified form of this is $f(x)=(x+1)\left(x+\frac{3}{2}\right)\left(x-\frac{7}{5}\right)$.

The rational zeros for $f(x)$ are $-1$, $-\frac{3}{2}$, and $\frac{7}{5}$.

Using linear factors, $f(x)=(x+1)\left(x+\frac{3}{2}\right)\left(x-\frac{7}{5}\right)$.